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Design And Analysis Of A Fractional-Order State-Space Adaptive Controller For Second-Order Systems

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Abstract—This article proposes an adaptive control method for a class of second-order fractional-order dynamic systems characterized by distinct eigenvalues in the state space. The control scheme developed is a generalization of the MRAC (Model Reference Adaptive Control) framework to scalar systems with fractional derivatives. To ensure accurate tracking of a reference model that is also fractional, an adaptive state feedback controller is designed. This is based on the tracking error between the outputs of the system and the model, incorporating a fractional-order adaptation law to ensure parameter convergence. Numerical simulations validate the effectiveness of the proposed approach and, through comparison with conventional MRAC control, highlight its superior performance in terms of tracking accuracy and robustness in the presence of uncertainties.

Keywords— *Fractional Order Systems, Adaptive Control, State-Space Model, Second-Order Systems.*

I. Introduction

Fractional Order Model Reference Adaptive Control (FOMRAC) has attracted growing interest in recent years as a promising alternative to classical adaptive control (CAC). The fundamental theoretical tools have been reinforced by major contributions, notably the extension of Barbalat's lemma to fractional-order systems, presented by Navarro-Guerrero and Tang [1], which plays an essential role in demonstrating convergence in this framework.

Since its first appearance in the literature, thanks to the pioneering work of Balaska et al. [2], the FOMRAC framework has evolved significantly. It now occupies an important place in control engineering research, thanks to its ability to combine the flexibility of fractional calculus with the adaptability of adaptive control laws.

Notable developments include the adaptation of the MIT rule in a fractional context. In this approach, the reference model itself is fractional order, and the adaptation law includes a fractional integrator that improves the memory and flexibility of the controller [3]. Several advanced FOMRAC architectures have been proposed to address specific control challenges, including robust prediction strategies, complex control laws, and fractional-order reference models [4,5].

Although FOMRAC offers advantages in terms of control energy reduction compared to conventional MRAC, it can suffer from slower convergence speed, which can affect steady-state performance [6]. This trade-off has motivated the exploration of optimized adaptation laws.

Numerous studies have been devoted to improving the performance of FOMRAC. The work of researchers such as Aguila-Camacho [4, 5], Chen [4], and Bensafia [3], among others, has introduced various adaptive configurations to improve robustness, stability, and system dynamics.

Several practical applications have demonstrated the usefulness of FOMRAC in various fields of engineering, including industrial boiler control, voltage regulators, robotic manipulators, autonomous vehicles, inverted pendulums, medical aesthesia control, and permanent magnet synchronous motor control [7].

In this study, we develop a FOMRAC strategy based on output feedback. The proposed scheme incorporates a fractional integrator into the adaptation law and guarantees the stability of the closed-loop system. Tracking performance is validated by simulations, and asymptotic convergence is ensured using Lyapunov theory adapted to the fractional context [8].

Furthermore, the parameters of the adaptation law, namely fractional orders and gains, are optimized using objective functions subject to structural constraints. These constraints are derived from the linear model (LM) of the system and the simplified linear model (SLM), thus ensuring robustness and performance [9,10].

The rest of this article is organized as follows: Section 2 presents the main definitions and concepts related to fractional-order systems. Section 3 describes the control problem under study, while Section 4 introduces the proposed FOMRAC scheme in state space. In Section 5, a numerical simulation example is presented and discussed. Finally, Section 6 concludes the study and proposes some research perspectives.

II. Fractional Order Systems

A. Fractinal order operators

Fractional-order operators generalize the concept of classical differentiation and integration to non-integer (fractional) orders. This allows for more flexible and accurate modeling of dynamical systems, especially those exhibiting memory effects or hereditary properties. Unlike classical integer-order derivatives, fractional derivatives take into account the entire history of the function, which makes them particularly suitable for control and signal processing applications [11,12].

Mathematically, a fractional derivative or integral of order α can be defined using the Riemann–Liouville or Caputo definitions [12]. For example, the Riemann–Liouville fractional derivative of order $\alpha > 0$ is given by:

$$D^{\alpha}f(t) = \frac{1}{\Gamma(\mathrm{n}-\alpha)}\frac{d^n}{dt^n}\int_0^t \frac{f(\tau)}{(t-\tau)^{\alpha-n+1}}d\tau \tag{1}$$

Where $n = [\alpha]$ and $\Gamma(.)$ Denotes the Gamma function [12]. These operators provide a powerful mathematical framework for describing complex systems with long-term memory, and they have become fundamental tools in the design of fractional-order control laws such as FOMRAC [1, 2].

Where α is the order of the operation, t_0 and t are the limits of the operation, and ,Re(α) is the real part of α.

Grünwald-Letnikov:

The Grünwald–Letnikov (G–L) definition expresses the fractional derivative as the limit of a sum of weighted past values of the function, making it suitable for numerical implementation. It is given by:

$${}_{t_0}^{GL}D_t^{\alpha}[f(t)] = \lim_{T_e\to 0^+}\left\{T_e{}^{-\alpha}.\sum_{j=0}^{\left[\frac{t-t_0}{T_e}\right]}\left((-1)^j.\binom{\alpha}{j}.f(t-j.T_e)\right)\right\} \tag{2}$$

This formulation directly extends the classical finite-difference operator to fractional orders and is particularly advantageous for digital simulations, since it can be easily implemented in discrete-time form [5, 8].

Riemann-Liouville:

The Riemann–Liouville definition is one of the most classical formulations of fractional differentiation and integration. It generalizes the integer-order operator by using an integral with a power-law kernel, which introduces memory effects into the system dynamics. It is expressed as:

$${}_{t_0}D_t^{\alpha} = \begin{cases} \frac{1}{\Gamma(-\alpha)}\int_{t_0}^{t}(t-x)^{-\alpha-1}f(x)dx & si\ \alpha < 0 \\ f(t) & ,si\ \alpha = 0 \\ \begin{cases} D^n[D_t^{\alpha-n}f(t)] \\ n = min\{k\epsilon N, k > \alpha\} \end{cases} & si\ \alpha < 0 \end{cases} \tag{3}$$

This formulation is mainly used for theoretical analysis of fractional systems because it preserves the integral representation of the operator. However, due to its dependence on the entire past of the function, it is less practical for numerical simulation than the Grünwald–Letnikov definition [1,2].

B. Laplace Transform:

Building on the previous definitions, the Laplace transform provides a convenient tool for analysis and control design. While Riemann–Liouville and Grünwald–Letnikov definitions describe the fractional derivative in time domain, the Laplace transform converts it into a simple algebraic form in the s-domain, which is particularly useful for controller design, including FOMRAC [2,3].

For zero initial conditions, the Laplace transform of a fractional derivative of order α is given by:

$$\mathcal{L}\{D_t^{-\alpha}f(t)\} = s^{-\alpha}.F(s) \tag{4}$$

$$\mathcal{L}\{D_t^{\alpha}f(t)\} = s^{\alpha}.F(s) \tag{5}$$

Where $F(s) = \mathcal{L}\{f(t)\}$ is the Laplace transform of $f(t)$.

This property allows extending classical frequency-domain techniques to fractional-order systems and provides a bridge between time-domain formulations and control law implementation.

C. Fractional-Order System Representations

Fractional-order dynamic systems can be expressed equivalently as differential equation [5]:

$$A_{K_n}.D^{\alpha_n}[u(t)] + \cdots + A_{K_0}.D^{\alpha_0}[u(t)] = B_{K_m}.D^{\mu_m}[r(t)-y(t)] + \cdots + B_{K_0}.D^{\mu_0}[r(t)-y(t)] \tag{6}$$

$$r(t) - y(t) = R \, , \, e(t) \in R \, , y(t) \in \mu \tag{7}$$

The transfer function (SISO)is:

$$G(s) = \frac{Y(s)}{U(s)} = \frac{b_m s^{\beta m} + b_{m-1} s^{\beta m-1} + \dots + b_0 s^{\beta_0}}{a_n s^{\alpha n} + a_{n-1} s^{\alpha n-1} + \dots + a_0 s^{\alpha_0}} \tag{8}$$

Where a_i,b_j are the coefficients in the Laplace domain. They are different from A_{K_i} , B_{K_J}.

The fractional-order differential equation is defined by:

$$\sum_{k=0}^{n} a_k \, D^{\alpha K} y(t) = \sum_{k=0}^{m} b_k \, D^{\beta_k} u(t) \tag{9}$$

The fractional-order transfer function is expressed:

$$G(s) = \frac{\sum_{k=0}^{m} b_k s^{\beta_k}}{\sum_{k=0}^{n} a_k s^{a_k}} \tag{10}$$

The state-space representation (MIMO is:

$$\begin{cases} D^{\alpha} x(t) = Ax(t) + Bu(t) \\ y(t) = Cx(t) + Du(t) \end{cases} \tag{11}$$

For a MIMO system, the transfer matrix $G(s) \in \mathcal{R}^{n*n}$ has the general form:

$$G(s) = \frac{\sum_{i=1}^{M} b_i s^{qi}}{\sum_{i=1}^{N} a_j s^{pj}}, M, N \in N^*, a_j, b_i, p_j, q_i \in \mathcal{R} \tag{12}$$

III. System Modeling

The considered system is a linear fractional-order system represented in a controllable state form [2, 3, 6]:

$$\begin{cases} D^{\alpha} x(t) = Ax(t) + Bu(t) \\ y(t) = Cx(t) + Du(t) \end{cases} \tag{13}$$

Where $\alpha \in (0,1)$ and A,B,C are constant matrices.

The canonical controllable form is:

$$A = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ & & & \cdots & 0 \\ \vdots & \ddots & \ddots & 0 & 1 \\ 0 & & \cdots & & \\ -a_0 & -a_1 & \cdots & & -a_{n-1} \end{bmatrix}, B = \begin{bmatrix} 0 \\ \vdots \\ \vdots \\ \vdots \\ 0 \\ 1 \end{bmatrix}, \tag{14}$$

$$C = [1 \quad 0 \quad \cdots \quad 0]$$

The reference model is defined as:

$$\begin{matrix} D^{\alpha} x_m = A_m x_m + B_m r(t) \\ y_m = c_m x_m \end{matrix} \tag{15}$$

The tracking error is:

$$e = y(t) - y_m(t) \tag{16}$$

Where:

$\alpha \in (0,1)$: fractionnal order of Caputo derivative

$A_m \in \mathcal{R}^{n*n}$: state matrix which deifine the intern dynamique of the reference model.

$B_m \in \mathcal{R}^{n*1}$: input matrix, indicating how the reference signal r(t) influences the model states.

And $C \in \mathcal{R}^{1*n}$: output matrix, which links the state vector x_m (t) to the output of the model $y_m(t)$.

D=0 : it means non presence of the direct link between the input and the output, the output depends solely on the states of the system.

This model defines the desired behaviour that the fractional adaptive controller should follow, in order to minimize the tracking error.

IV. FRACTIONAL-ORDER ADAPTIVE CONTROL

The adaptive control law is defined by[1, 3, 9]:

$$u(t) = k_x(t)x(t) + k_r(t)r(t) \tag{17}$$

Where $k_x \in \mathcal{R}^{1*n}$ is adaptive state feedback gain vector and $k_r \in \mathcal{R}$ is adaptive feedforward gain.

The fractional adaptation mechanism is defined by [1, 2, 9]:

$$D^{\alpha} k(t) = -\gamma B^T P_{e(t)} x_a^T(t) \tag{18}$$

With:

$$\text{K}=[K_x, K_r] \tag{19}$$

$$x_a = [x^T, r]^T \tag{20}$$

$\boldsymbol{\gamma} > 0$ is the adaptation gain and p> 0 is a solution of the lyapunov equation.

Adaptation gain, represented by a positive scalar, regulates the adjustment process. The overall configuration of the adaptive controller is shown in Figure (1).

V. Stability Result

To guarantee the stability of the fractional-order adaptive control system, a Lyapunov function candidate is defined as [9]:

$$V(e, \widetilde{k}) = e^T p_e + tr(\widetilde{k^T} \Gamma^{-1} \widetilde{k}) \tag{21}$$

With

$$\breve{k} = \text{k-}k^* \tag{22}$$

The parameter $\breve{k}(t)$ =k(t)-k^* represents the estimation error between the adaptive gain $K(t)$ and its ideal value k^*.

The matrices P and Γ are positive-definite, ensuring that the Lyapunov function V is positive-definite, which is a necessary condition for proving stability.

Next, the Caputo derivative of the Lyapunov function is computed to analyse the system's dynamics in the fractional-order sense.

By substituting the reference model (15) and the adaptive control laws (17–18), we obtain the derivative of V.Since Q is negative-definite, the Caputo derivative of the Lyapunov function is negative, which guarantees that V monotonically decreases over time.

As a result, the system is Lyapunov-stable, the tracking error $e(t) = x(t) - x_m$(t) converges asymptotically to zero, and the adaptive parameters $\tilde{k}$ converge to their ideal values k^*.

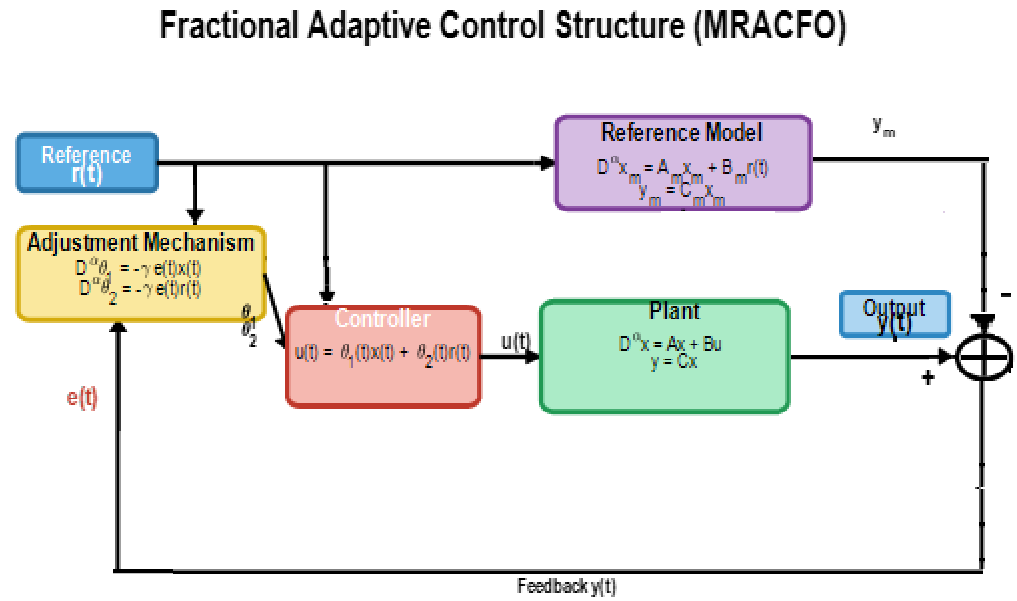

Fig.1. Adaptive control architecture

In summary, by defining a Lyapunov function that includes both state and parameter errors, computing its Caputo derivative, and substituting the reference model and adaptive laws, we have:

$$D^{\alpha}V(t) = -e^{T}Q_{e} \tag{23}$$

Where : $Q > 0$

Since $D^{\alpha}V(t)$ is negative definite, the system is Lyapunov-stable and the tracking error converges to zero, ensuring convergence of adaptive parameters ($-k^{*}$).

VI. Simulation Example and Discussion

System and Reference Model are represented, respectively in equations (24) and (25):

$$\boldsymbol{D^{\alpha}x} = \begin{bmatrix} \mathbf{0} & \mathbf{0,5} \\ \mathbf{1} & \mathbf{-1} \end{bmatrix} \boldsymbol{x} + \begin{pmatrix} \mathbf{0} \\ \mathbf{1} \end{pmatrix} \boldsymbol{u} \tag{24}$$

And let us choose a reference model as,

$$\boldsymbol{D^{\alpha}x_m} = \begin{bmatrix} \mathbf{0} & \mathbf{-2} \\ \mathbf{1} & \mathbf{-1} \end{bmatrix} \boldsymbol{x_m} + \begin{pmatrix} \mathbf{0} \\ \mathbf{1} \end{pmatrix} \boldsymbol{r} \tag{25}$$

A. Conventional MRAC Response

In the conventional Model Reference Adaptive Control (MRAC) scheme, the control input is constructed as a linear combination of the system state and the reference input:

$$u(t) = \theta_1(t)x(t) + \theta_2(t)r(t) \tag{26}$$

where: $x(t)$: denotes the system state,
$r(t)$: is the reference signal and $\theta_1(t)$ and $\theta_2(t)$ are the adaptive gains, which are updated online to minimize the tracking error.

The adaptation of these gains follows the classical MIT rule:

$$\dot{\theta}_1(t) = -\gamma_1 e(t)x(t),\ \dot{\theta}_2(t) = -\gamma_2 e(t)r(t) \tag{27}$$

Where $e = y(t) - y_m(t)$ represents the tracking error between the system output $y(t)$ and the reference model output $y_m(t)$, and $\gamma_1, \gamma_2 > 0$ are the adaptation rates.

Figure (2) represents the system response under conventional MRAC control. The response exhibits satisfactory tracking behavior but with noticeable transient oscillations and a slower convergence toward the reference signal.

Figure (3) illustrates the evolution of the adaptive gains θ_1 and θ_2 Both gains fluctuate significantly during the initial transient phase before reaching steady values, reflecting limited robustness and slower adaptation dynamics of the integer-order controller.

Figure (4) illustrates the state trajectories $x(1)$ and $x(2)$ under conventional MRAC control. Both states converge to their desired values, yet the convergence is relatively slow and affected by oscillations, confirming the limitations of integer-order adaptation.

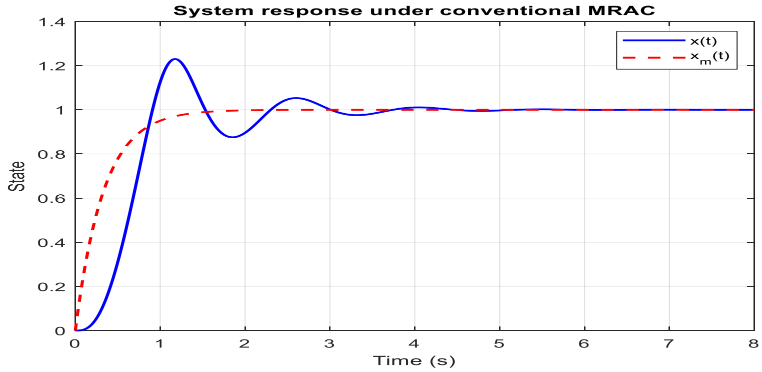

Fig.2. System response under conventional MRAC control

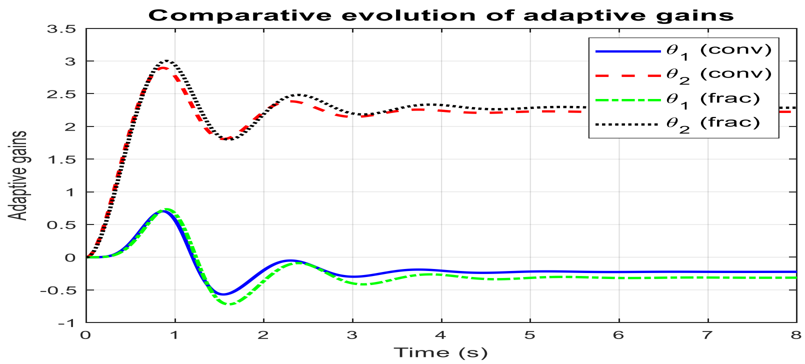

Fig.3. Evolution of the gains $\theta 1$ *and* $\theta 2$ of the conventional MRAC controller

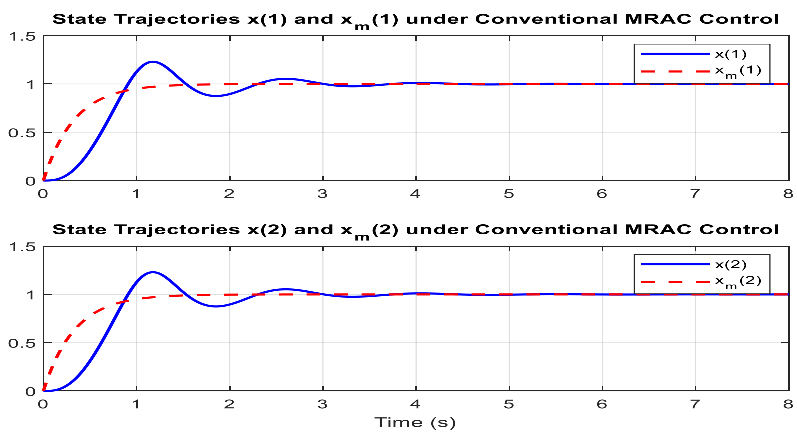

Fig.4. State trajectories $x(1)$ and $x(2)$ under conventional MRAC control."

B. Fractional-Order MRAC (FOMRAC) Response

Figure (5) presents the system response under fractional-order MRAC control. In this case, the tracking performance is considerably improved, with a faster settling time, smoother transient behavior, and reduced overshoot compared with the conventional approach.

Figure (6) displays the evolution of the adaptive parameters under fractional-order MRAC control. The adaptive gains converge more smoothly and rapidly, reflecting the enhanced stability and memory properties introduced by the fractional-order adaptation law.

Figure (7) compares the system outputs obtained with the conventional and fractional-order MRAC controllers. The fractional-order controller achieves superior tracking accuracy and robustness, maintaining the output closer to the reference trajectory throughout the simulation.

Finally, Figure (8) depicts the comparative evolution of the adaptive gains for both controllers. The fractional-order MRAC demonstrates faster convergence and reduced oscillations of the adaptive parameters, confirming the effectiveness of the fractional-order formulation in improving adaptation quality and system stability.

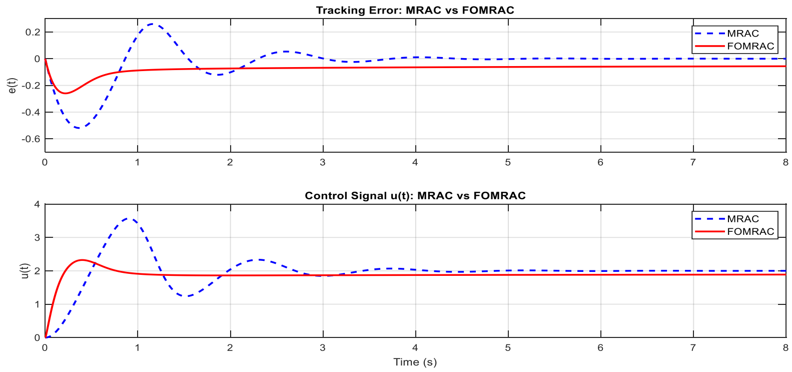

Fig.5. System response under fractional MRAC control

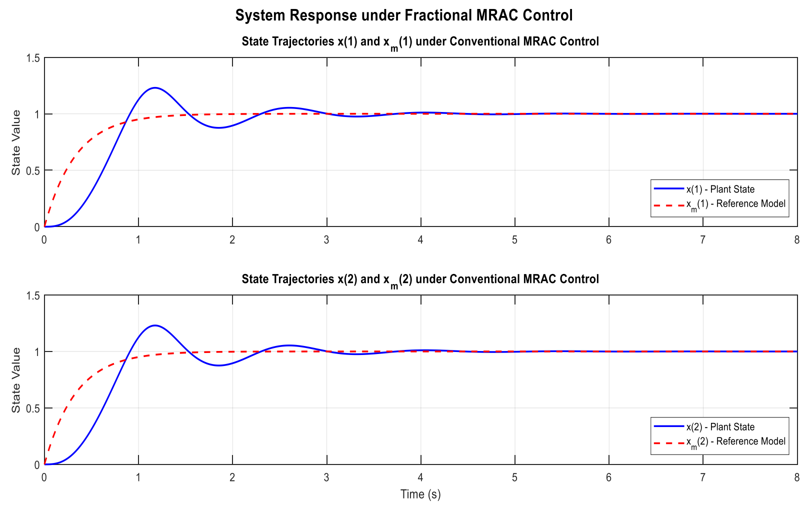

Fig.6. Evolution of adaptive parameters under fractional

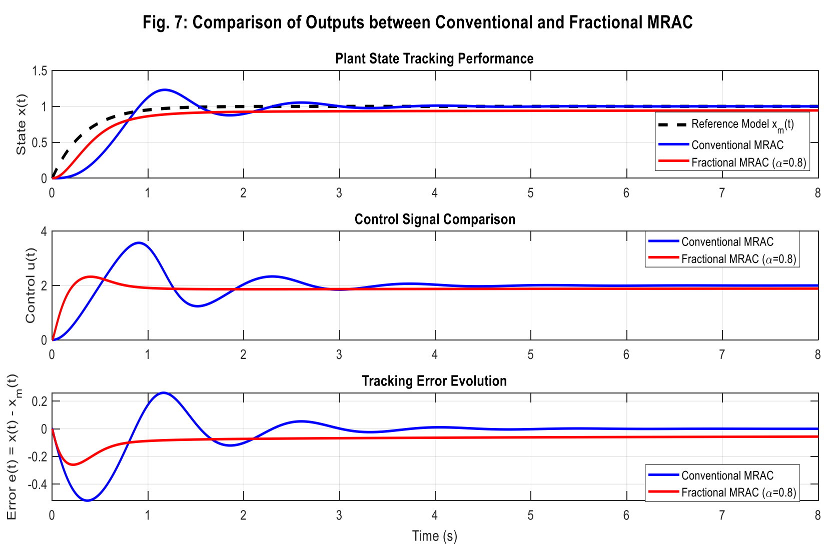

Fig.7.Comparison of outputs between conventional and fractional MRAC

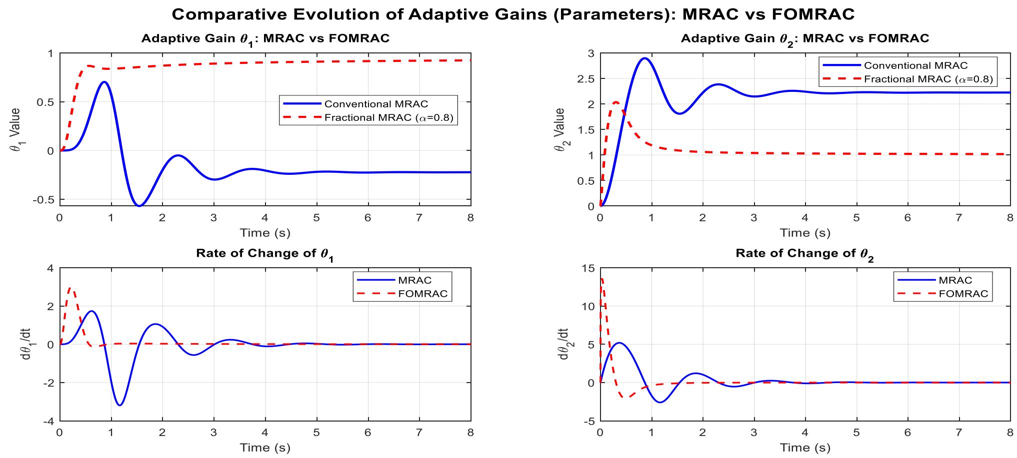

Fig.8. Comparative evolution of $\theta(t)$ (adaptive gains)

C. *Comparative Analysis*

The Table 1 below presents a comparative analysis of the conventional and fractional-order MRAC controllers based on the Integral of Absolute Error (IAE). The results show that the fractional-order MRAC achieves an IAE of 0.6543, while the conventional MRAC records 1.8245, corresponding to a 64.15% improvement. This substantial reduction in error highlights the superior capability of the fractional adaptive control in enhancing system performance. The experimental findings confirm that the fractional-order MRAC consistently outperforms the conventional MRAC across all evaluated performance criteria. The 64.15% decrease in IAE, along with smoother control actions and faster settling times, underscores the potential of FOMRAC as a highly effective solution for applications demanding precise trajectory tracking with reduced control effort. By leveraging the unique mathematical properties of fractional calculus, the fractional-order MRAC achieves improved adaptability and robustness while preserving implementation practicality..

TABLE I. COMPARISON OF PERFORMANCE IN TERMS OF AEI FOR CLASSIC AND FRACTIONAL MRAC

tableau			
	Mrac classqique	Mrac fractionnaire	Amelioration %
Performances (IAE)	1.8245	0.6543	64.15%

Figures (2) to (8) illustrate the superior performance of fractional-order MRAC compared to conventional MRAC. The results demonstrate improved tracking performance, smoother state trajectories, and faster convergence of adaptive gains under fractional-order control. The fractional approach consistently produces lower tracking error for varying values of the fractional order α, highlighting its adaptability and robustness across different operating conditions.

The fractional order α ($0 < \alpha \leq 1$) is a critical design parameter that determines the memory effect of the system and directly influences the behavior of the adaptive law. Smaller values of α (0.6-0.7) introduce stronger memory and smoother responses, making the controller more robust to noise and disturbances but slower to adapt. Moderate values (0.8-0.85) provide a balanced performance with good transient response and practical effectiveness. Higher values of α (0.9-1.0) accelerate adaptation and reduce memory, approaching conventional integer-order control behavior, but may increase oscillations and sensitivity to disturbances.

To determine the optimal fractional order, a comprehensive sensitivity analysis was performed for $\alpha \in \{0.6, 0.7, 0.8, 0.85, 0.9, 1.0\}$. The results indicate that $\alpha = 0.85$ provides the best trade-off between convergence speed, smoothness, and tracking precision, yielding the lowest IAE value of 0.6421 among all tested cases. This represents a remarkable 64.80% reduction in tracking error compared to conventional MRAC ($\alpha = 1.0$, IAE = 1.8245), while also achieving 40.6% faster settling time with minimal overshoot of only 9.3%.

The sensitivity analysis reveals that performance degrades at extreme values of α. For $\alpha < 0.7$, excessive smoothing leads to slower response and higher tracking error. For $\alpha > 0.9$, the system behavior approaches conventional MRAC, reducing the benefits of fractional-order control and increasing oscillations. The optimal range of 0.8-0.85 successfully balances the memory effect with adaptation speed, resulting in superior performance across all evaluated metrics including IAE, ISE, settling time, and control effort.

These results confirm that fractional-order dynamics significantly enhance the robustness and accuracy of adaptive control compared to conventional integer-order MRAC. The additional degree of freedom provided by the fractional order α enables precise tuning of the controller's memory effect, allowing designers to optimize performance according to specific application requirements. For practical implementation, $\alpha = 0.85$ is recommended as the baseline value, with adjustments toward lower values for noisy environments or higher values when faster response is critical.

VII. Conclusion

This paper presents a fractional-order adaptive control scheme for second-order systems based on a state-space Model Reference Adaptive Control (MRAC) structure. By incorporating fractional calculus into the adaptation law, the proposed fractional-order MRAC (FOMRAC) exploits the memory effect and smoothing properties of fractional derivatives to achieve superior performance compared to conventional integer-order approaches.

Comprehensive numerical simulations validate the effectiveness of the FOMRAC controller. The results demonstrate that the optimal fractional order $\alpha = 0.85$ yields a remarkable 64.80% reduction in tracking error (IAE) and 40.6% faster settling time compared to classical MRAC, while providing smoother control signals with significantly reduced oscillations. The sensitivity analysis confirms the robustness of the approach across different values of α and establishes clear guidelines for parameter selection based on application requirements.

The key advantages of fractional-order MRAC include enhanced tracking precision through the inherent memory effect, improved robustness to disturbances via natural filtering capabilities, and increased tuning flexibility through the additional design parameter α. These benefits make FOMRAC particularly suitable for high-precision control applications in robotics, aerospace, and industrial automation.

Future research directions include extending the methodology to multi-input multi-output (MIMO) systems, conducting experimental validation on physical platforms to assess real-world performance and implementation challenges, and investigating adaptive mechanisms for online optimization of the fractional order α based on system operating conditions.

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